

Bulk Viscosity in Hybrid Stars

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We compute the bulk viscosity of a mixed quark-hadron phase. In the first scenario to be discussed, the mixed phase occurs at large densities and we assume that it is composed of a mixing of hyperonic matter and quarks in the Color Flavor Locked phase. In a second scenario, the mixed phase occurs at lower densities and it is composed of a mixing of nucleons and unpaired quark matter. We have also investigated the effect of a non-vanishing surface tension at the interface between hadronic and quark matter. In both scenarios, the bulk viscosity is large when the surface tension is absent, while the value of the viscosity reduces in the second scenario when a finite value for the surface tension is taken into account. In all cases, the r-mode instabilities of the corresponding hybrid star are suppressed.

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I. INTRODUCTION

The discovery by Andersson, Friedman and Morsink of r-mode instabilities in neutron stars put rather severe limits on the highest rotation frequency of pulsars [1, 2]. These constraints can be incompatible with the existence of millisecond pulsars, if the instability is not suppressed by a sufficiently large viscosity. Actually, for a star composed only of neutrons and protons and for temperatures larger than roughly 10^{10} K, the bulk viscosity due to the modified Urca process is large enough to damp the instability [3]. On the other hand, when the star cools down to lower temperatures, the instability is not suppressed and the star is forced to loose angular momentum via emission of gravitational waves [4]. More recently, it has been noticed that the existence of viscous boundary layers at the interface between the fluid core and the crust can stabilize the star [29] if the rotation period is longer than ~ 1.5 ms [5, 6]. At very low temperatures, below 10^8 K, shear viscosity becomes large and it allows older stars to increase their angular velocity by mass accretion.

Actually, compact stars can be constituted by a larger variety of particles than just neutrons and protons. One possibility is that hyperons form at the center of the star. It has been shown that, due to non-leptonic weak reactions, bulk viscosity can be rather large for an hyperonic star [7, 8], which therefore can emit gravitational waves only if its temperature is $\sim 10^{10}$ K and its frequency is larger than 10–30% of its Keplerian frequency.

The formation of quark matter inside a compact star has been discussed extensively in the literature. Bulk viscosity of non-interacting strange quark matter is very large [9, 10, 11]. On the other hand, recent studies taking into account quark-quark interaction revealed the possible existence of color superconducting phases in which quarks form Cooper pairs with gaps as large as 100 MeV [12]. In that case, bulk viscosity is strongly suppressed by the large energy gaps, r-mode instabilities are not damped and pure color-superconducting quark stars seem therefore to be ruled out by the pulsar data [13].

In the present paper we are interested in studying the

viscosity and stability, respect to r-modes, of an Hybrid Star (HyS), for which no quantitative analysis has been performed so far. If the star is made of hyperonic matter and of non-interacting quarks, it is rather obvious that the viscosity of the HyS should be large, due to the large viscosity of its constituents. The real question concerns the case in which color-superconducting quark matter is considered, since its viscosity is negligible. There are in principle (at least) two sources of viscosity for HySs. One originates at the interface between the crust and the fluid interior [13] and it is similar to the one discussed above in the case of purely hadronic stars [5, 6]. In our work we will not discuss that possibility and we will instead concentrate on a quantitative evaluation of the bulk viscosity in a mixed phase (MP) composed either of hyperons and color-superconducting quark matter or of nucleons and non-interacting quarks. We have also investigated the effect of the existence of finite-size structures in the MP. These geometrical structures can exist if the surface tension at the interface between hadronic and quark matter is not vanishing. We will show that the bulk viscosity of the MP is in general rather large, particularly so when the surface tension is negligible. Therefore HySs are possible candidates for young millisecond pulsars.

II. EQUATION OF STATE

We construct the Equation of State (EOS) of matter at high density modeling the Hadronic phase by a relativistic non-linear Walecka type model [14] with the inclusion of Hyperons [15]. Concerning quark matter, we use an MIT-bag like EOS in which quarks can pair to form a Color Flavor Locked (CFL) phase [16, 17, 18]. CFL is considered to be the energetically favored type of pairing pattern, at large densities, in the case of β -stable, electric-charge neutral quark matter. At lower densities, a two-flavor Color Superconducting (2SC) phase can form, with a smaller energy gap [19].

Two scenarios are possible, depending on the value of the critical density separating hadronic matter from MP.

In the first case the critical density is large, therefore hyperons start being produced at a density smaller than the critical one. We will call this phase “hyperon-quark” MP [20]. This is the first case discussed below and it corresponds to the upper panel of Fig. 1. In the second scenario, the MP starts at a density lower than the hyperonic threshold and the hyperon density is much smaller than in the previous case. We will explore in particular the situation in which hyperons are completely eaten-up by quarks and they do not appear in the “nucleon-quark” MP (see lower panel of Fig. 1). In this second scenario we will assume that in the MP the density is so low that the CFL pairing cannot form and, for simplicity, we will consider unpaired quarks in the MP, since anyway the bulk viscosity of the 2SC quark phase is similar to the viscosity of unpaired quark matter [13]. In both scenarios, we describe the composition of the MP assuming a first order transition and imposing Gibbs conditions [30]. Beta stability and charge neutrality are satisfied in every density region.

III. BULK VISCOSITY

Bulk viscosity is the dissipative process in which a perturbation of the pressure of a fluid element is converted to heat. A small variation of the pressure of the system can be treated as a perturbation on the densities of the different species of particles bringing the system out of β -equilibrium. The reactions between the different particles drive the system back to an equilibrium configuration, with a delay which depends on the characteristic time scale of the interactions. Following the formalism of Ref.[8], we classify all the reactions either as fast or slow. Slow processes produce bulk viscosity, because their time scale is comparable with the period of the perturbation. Fast processes, instead, put constraints on the variation of the densities of the different particles. In the following we generalize the formalism of Ref. [8] in order to compute the bulk viscosity of MP. The equations needed in the computation are rather different in the two scenarios discussed above and we will have to deal with all the different cases separately. Moreover, if the effect of a non-vanishing surface tension is taken into account, the formalism needed to compute the bulk viscosity has to be further modified. It is well known that a finite value for the surface tension allows the formation of finite size structures whose geometrical shape is determined by the interplay of the various contribution to the energy, including the Coulomb term [21, 22]. A precise estimate of surface tension σ is unfortunately still lacking, and values ranging from a few MeV/fm² to a few tens MeV/fm² have been discussed in the literature. We can roughly divide this range of values in three windows. Values larger than ~ 30 MeV/fm² would not allow the formation of the mixed phase since it would not be energetically favored[17, 22]. Our analysis is therefore restricted to smaller values. In the following we will discuss both the

case in which the surface tension is so small that it can be neglected and the case in which it is non-vanishing.

A. Negligible surface tension

Let us first discuss the case in which the surface tension is so small that the perturbations of the star, associated e.g. to r-modes, can break the finite-size structures present in the MP. We have estimated that this case corresponds to values of the surface tension $\sigma \lesssim 1$ MeV/fm², since to r-modes excitations is associated an energy per baryon of a few MeV.

1. First scenario

We start our analysis from the first scenario discussed in Sec. II, namely from the case in which hyperons are present in the MP. We will discuss in particular a parameter set in which only Λ and Σ^- particles are produced in the MP. The only slow processes generating viscosity are non-leptonic reactions between hadrons [31], since the large gap prevents weak reactions between quarks. As in Ref.[8], we consider the reactions

$$n + n \xleftrightarrow{H_W} p + \Sigma^-, \quad (1)$$

$$n + p \xleftrightarrow{H_W} p + \Lambda. \quad (2)$$

Following [8], we have not taken into account the reaction $n + n \xleftrightarrow{H_W} n + \Lambda$, since the corresponding reaction rate cannot be easily estimated. In Ref.[7] it has been argued that this rate can be one order of magnitude larger than the one associated with reaction (2). In the first scenario, our results have therefore to be considered as upper limits for the viscosity, as it will be clarified in the following. Notice anyway that, concerning the damping of r-modes instabilities, to be discussed in the following, the most important region of the star corresponds to low-moderate densities, while the Λ is produced at larger densities as shown in Figs. (1) and (5).

Concerning fast processes, they come from the following reactions mediated by the strong interaction

$$n + \Lambda \xleftrightarrow{H_S} p + \Sigma^-, \quad (3)$$

$$\Lambda + \Lambda \xleftrightarrow{H_S} 2(uds)_{CFL}. \quad (4)$$

The last process describes the only possible “melting” of hadrons into CFL phase, since the pairing forces the number of up, down and strange quarks to be equal. This process is fast because of the vanishing value of σ . As we will see later the “melting” process is instead forbidden if the value of σ is not negligible. Concerning mechanical equilibrium, elastic scattering due to strong interactions, as well as melting processes like the one described in Eq.(4), are responsible for (rapid) momentum transfer between the two phases. The mechanical equilibrium

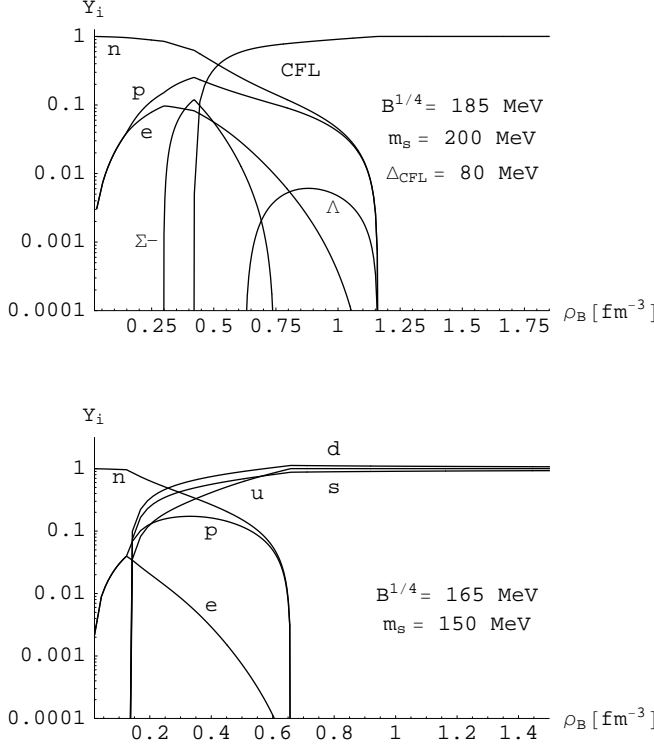


FIG. 1: Particle abundances, as function of the total baryon density. The upper panel corresponds to the case of hyperon-quark MP and the lower panel to nucleon-quark MP.

between the two phases is reached in a time scale t_s determined by strong interaction, $t_s \sim 10^{-23}$ s. The full system is, instead, out of mechanical equilibrium on a time scale of the order of the period of the perturbation $t_p \sim 10^{-3}$ s. Therefore, during a fluctuation the two components of the fluid remain in mutual mechanical equilibrium. The variations of the pressure in the two phases have therefore to satisfy the constraint:

$$\delta P_h = \delta P_{CFL}. \quad (5)$$

The variations $\delta\rho_i$ of the densities of the various particles and the variation $\delta\chi$ of the quark fraction are constrained by the following linearized equations

$$0 = (1 - \chi)(\delta\rho_n + \delta\rho_p + \delta\rho_\Lambda + \delta\rho_\Sigma) + \chi(\delta\rho_q) + \delta\chi(\rho_q - \rho_n - \rho_p - \rho_\Lambda - \rho_\Sigma), \quad (6)$$

$$0 = (1 - \chi)(\delta\rho_p - \delta\rho_\Sigma) - \delta\chi(\rho_p - \rho_\Sigma), \quad (7)$$

$$0 = \sum_{\{H\}} p_H \delta\rho_H - p_q \delta\rho_q, \quad (8)$$

$$0 = \beta_n \delta\rho_n + \beta_p \delta\rho_p + \beta_\Lambda \delta\rho_\Lambda + \beta_\Sigma \delta\rho_\Sigma, \quad (9)$$

$$0 = \alpha_{\Lambda n} \delta\rho_n + \alpha_{\Lambda p} \delta\rho_p + \alpha_{\Lambda\Lambda} \delta\rho_\Lambda + \alpha_{\Lambda\Sigma} \delta\rho_\Sigma - 3\alpha_{qq} \delta\rho_q, \quad (10)$$

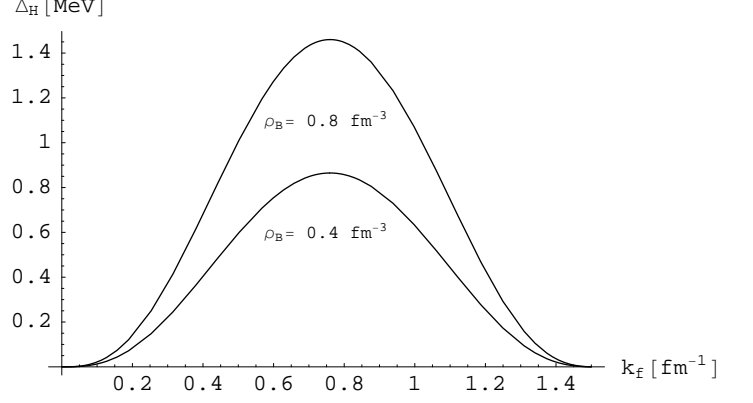


FIG. 2: Hyperon superfluid gap at zero temperature and for two values of the baryon density. (see Ref. [8])

where

$$\alpha_{ij} = (\partial\mu_i/\partial\rho_j)_{\rho_k, k \neq j}, \quad (11)$$

$$\beta_i = \alpha_{ni} + \alpha_{\Lambda i} - \alpha_{pi} - \alpha_{\Sigma i}, \quad (12)$$

$$p_i = \partial P / \partial \rho_i. \quad (13)$$

Eqs. (6),(7) impose baryon number conservation and electric charge neutrality. Eq. (8) (where the sum runs on all hadrons) imposes the mechanical equilibrium defined by Eq. (5). Finally, Eqs. (9),(10) describe the equilibrium respect to the two strong processes of Eqs. (3),(4). Notice that $\delta\chi$ does not appear in Eqs. (8)–(10), because neither the pressure nor the chemical potentials explicitly depends on the quark volume fraction χ . Solving the system allows to express all the $\delta\rho_i$ and $\delta\chi$ as function of $\delta\rho_n$ [32].

The relaxation time τ associated to the weak processes reads

$$\frac{1}{\tau} = \left(\frac{\Gamma_\Lambda}{\delta\mu} + 2 \frac{\Gamma_\Sigma}{\delta\mu} \right) \frac{\delta\mu}{\delta\rho_n}. \quad (14)$$

Here Γ_Λ and Γ_Σ are the rates of the weak interactions [33]. They have been calculated in Eqs. (4.21), (4.28), (4.29) of Ref.[8], where it has been shown that the main dependence of the rate on the temperature is a quadratic one. Moreover, if hyperon superfluid gaps as the ones displayed in Fig. 2 do develop, they exponentially suppress the rates as it will be discussed later.

The chemical potential unbalance $\delta\mu$ is given by

$$\delta\mu \equiv \delta\mu_n - \delta\mu_\Lambda = 2\delta\mu_n - \delta\mu_p - \delta\mu_\Sigma, \quad (15)$$

where the variations of the chemical potentials can be expressed in terms of $\delta\rho_i$ as $\delta\mu_i = \sum_j \alpha_{ij} \delta\rho_j$. A unique $\delta\mu$ appears in Eq. (15), since the variations $\delta\mu_\Lambda$ and $\delta\mu_\Sigma$ are constrained by the fast process given in Eq. (3). The real part of bulk viscosity, which is the relevant quantity

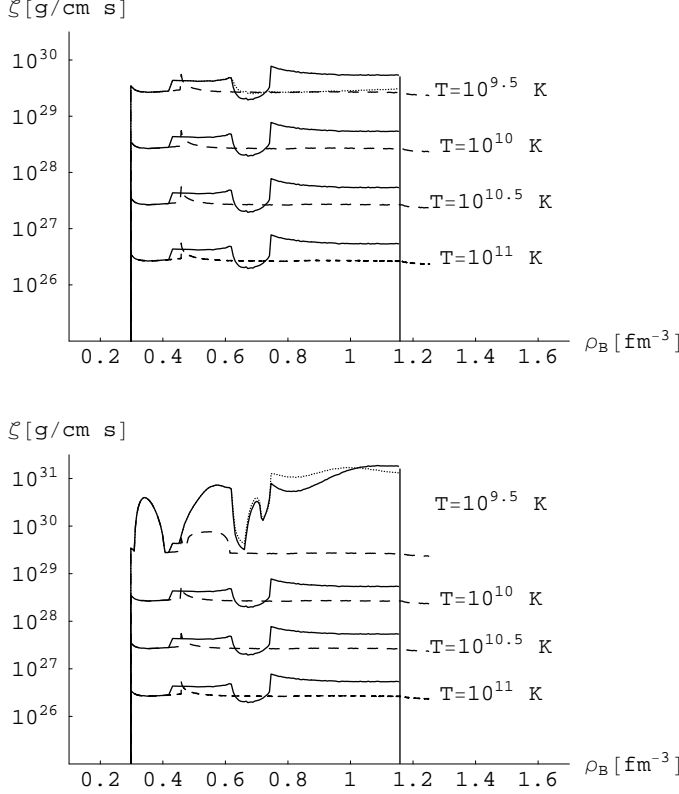


FIG. 3: Bulk viscosity in the first scenario as a function of baryon density for various temperatures. Solid lines refer to MP viscosity, while dashed lines correspond to pure hyperonic matter (vanishing hyperon gap in the upper panel and finite hyperon gap in the lower panel). The dotted lines corresponds to the case in which a large hadron-quark surface tension ($10 \text{ MeV/fm}^2 < \sigma < 30 \text{ MeV/fm}^2$) is taken into account and they are computed for $T = 10^{9.5} \text{ K}$ (see text).

for damping r-modes instabilities, finally reads

$$\zeta = \frac{P(\gamma_\infty - \gamma_0)\tau}{1 + (\omega\tau)^2}, \quad (16)$$

where γ_∞ and γ_0 are the “infinite” frequency adiabatic index and the “zero” frequency adiabatic index and ω is the angular velocity of the perturbation.

It is interesting to remark that there are two asymptotic behaviors of the viscosity as a function of the frequency ω . In the high frequency case ($\omega\tau \gg 1$), the viscosity scales as $1/\tau$, while in the low frequency limit the viscosity is proportional to τ . In the parameter and temperature ranges explored in our paper it results that we always remain in the low frequency limit, as it can be seen from Fig. 3 noticing that the viscosity decreases with the temperature. In this regime the addition of another weak decay (e.g. $n + n \xleftrightarrow{H_W} n + \Lambda$) decreases the

relaxation time and as consequence the viscosity too. In this sense our results for the viscosity in the first scenario must be considered as upper limits.

We can observe from Fig. 3 that, in the “hyperon-quark” MP scenario discussed so far, MP bulk viscosity is comparable to bulk viscosity of purely hyperonic matter even though the density of hyperons is very small in the MP (see upper panel of Fig. 1) [34]. Notice that in the small window of baryonic densities in which both Σ and Λ hyperons are present, the value of bulk viscosity of the MP and of the pure hadronic phase are essentially identical. On the other hand, in the baryonic density windows in which only one species of hyperons are present in the MP, the bulk viscosity of MP is larger than the bulk viscosity of the pure hadronic phase because only one decay channel is open (Σ decay of Eq. (1) or Λ decay of Eq. (2)) and therefore the relaxation time is larger. It is interesting to compute the bulk viscosity in this scenario also including the effect of hyperon superfluidity, as done in Ref. [8]. If the energy gaps Δ_H associated with the hyperons are non vanishing, the decay rates are suppressed by the factor $e^{-\Delta_H/T}$ where the gaps have a typical shape shown in Fig. 2. For low temperatures the viscosity displays the characteristic features shown in the lower panel of Fig. 3, while these features disappear at larger temperatures or for vanishing Δ_H . Concerning the appreciable difference between the viscosity of MP and of pure hadronic matter for $T = 10^{9.5} \text{ K}$, this is due to the Fermi momentum dependence of Δ_H , which implies that the gaps are suppressed for large hyperon densities. In the MP, the density of hyperons is lower than in pure hadronic matter and therefore the effect of the gaps shows out more dramatically.

Let us briefly discuss another possible source of bulk viscosity related to the formation of a boson condensate. Since all quarks in the CFL phase are gapped, the low energy excitations are the Nambu-Goldstone bosons associated with the spontaneous symmetry breaking of the global symmetries [16]. In particular, a condensate of π^- or K^- can appear, in the MP, as proposed in Ref. [18]. The role played by these boson condensates in the cooling of a CS has been studied in Ref. [23] and the condensates can play a role also in the calculation of the bulk viscosity of the MP. The weak processes involving these bosons are:

$$\pi^- \xleftrightarrow{H_W} e^- + \bar{\nu}_e, \quad (17)$$

$$K^- \xleftrightarrow{H_W} e^- + \bar{\nu}_e. \quad (18)$$

In the absence of these bosons, the variation of the density of electrons $\delta\rho_e$ can be neglected at low temperature ($T < 10^{10} \text{ K}$) because, as already remarked, all leptonic reaction rates are much smaller than those associated with non-leptonic reactions. If, on the other hand, the decay channels of Eqs. (17),(18) are open, then the corresponding decay rates are of the same order of the rates of the non-leptonic processes of Eqs. (1),(2) and, therefore, a new contribution to the viscosity appears. While

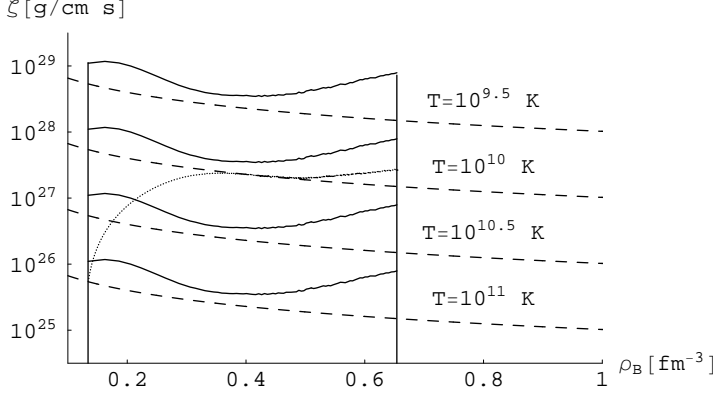


FIG. 4: Bulk viscosity as a function of baryon density, for various temperatures in the second scenario. Solid lines refer to MP viscosity, while dashed lines correspond to pure quark matter. The dotted line corresponds to the case in which a large nucleon-quark surface tension ($10 \text{ MeV/fm}^2 < \sigma < 30 \text{ MeV/fm}^2$) is taken into account and it is computed for $T = 10^{9.5} \text{ K}$ (see text).

a detailed calculation is clearly needed, the main result of our work, namely the existence of a large viscosity for $T \lesssim 10^{10} \text{ K}$, would even be strengthened.

2. Second scenario

We now discuss the second scenario in which a “nucleon-quark” MP forms, hyperons are absent and CFL quark pairing cannot take place in the MP [35]. We will assume that in the pure quark matter phase CFL gaps can form so that this phase will not contribute to the viscosity. The only source of viscosity, neglecting as before semi-leptonic reactions, is therefore

$$d + u \xleftrightarrow{H_W} u + s. \quad (19)$$

Concerning fast processes, they are given by the “melting” of nucleons into unpaired quarks. The chemical equilibrium respect to these reactions must therefore be satisfied during the perturbation. The linearized equations governing the density fluctuations, analogous to Eqs. (6)-(10) of the first scenario, read

$$0 = (1 - \chi)(\delta\rho_n + \delta\rho_p) + \chi(\delta\rho_u + \delta\rho_d + \delta\rho_s)/3 + \delta\chi((\rho_u + \rho_d + \rho_s)/3 - \rho_n - \rho_p), \quad (20)$$

$$0 = (1 - \chi)\delta\rho_p + \chi(2\delta\rho_u - \delta\rho_d - \delta\rho_s)/3 + \delta\chi((2\rho_u - \rho_d - \rho_s)/3 - \rho_p), \quad (21)$$

$$0 = \sum_{\{H\}} p_H \delta\rho_H - \sum_{\{Q\}} p_Q \delta\rho_Q, \quad (22)$$

$$0 = \alpha_{pn}\delta\rho_n + \alpha_{pp}\delta\rho_p - 2\alpha_{uu}\delta\rho_u - \alpha_{dd}\delta\rho_d, \quad (23)$$

$$0 = \alpha_{nn}\delta\rho_n + \alpha_{np}\delta\rho_p - \alpha_{uu}\delta\rho_u - 2\alpha_{dd}\delta\rho_d. \quad (24)$$

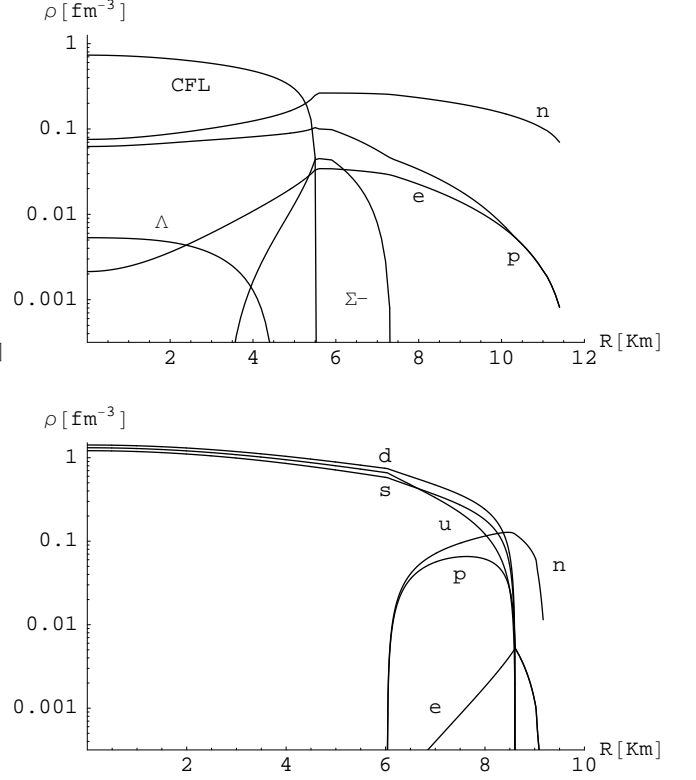


FIG. 5: Particles densities profiles inside hybrid stars of mass $M = 1.46 M_\odot$ in the two discussed scenarios. Parameters as in Fig. 1.

Eqs. (20)–(22) impose baryon number conservation, electric charge neutrality and mechanical equilibrium as in the first case. Eqs. (23),(24) describe the chemical equilibrium respect to the two fast processes of “melting”. As before we can calculate the chemical imbalance associated to the weak reaction (19) and the corresponding relaxation time. The rate of the process (19) is taken from Eqs. (5)–(7) of Ref.[10](see also footnote [36]). The resulting bulk viscosity is shown in Fig. 3 (lower panel). The bulk viscosity of “nucleon-quark” MP is of the same order of magnitude of bulk viscosity of strange matter [11].

B. Effects of the surface tension

Let us shortly discuss the effect of a non-vanishing surface tension σ at the interface between hadronic and quark matter. We will assume that $\sigma \lesssim 30 \text{ MeV/fm}^2$, so that MP can form because it is energetically favored. Finite-size structures (drops, rods, slabs) form at different densities, to minimize the energy of the MP. In this case the perturbation of the pressure due to r-modes is too weak to induce the “melting” process which is now

a very slow process (in comparison to the period of the perturbation) and it plays no role in the calculation of the bulk viscosity. The response of the finite size structures inside MP to a perturbation of the density corresponds to three possible processes: a) the formation of a drop of “new” phase; b) the merging of two structures into a single larger structure; c) the absorption of “old” phase into a structure of “new” phase. Obviously, also the reverse processes are possible. It is easy to see that large values of σ suppress all these processes. In particular, in case a) the radius of a critical drop of new phase increases with σ , making it more difficult to produce a new drop; case b) shares some similarity with the fission problem in nuclear physics, where the process of the separation of a heavy nucleus into two lighter nuclei is suppressed for larger values of σ since, during the fission (or merging) process, configurations having a large surface are produced. Finally, c) can be viewed as a special case of b), in which the absorbed hadron can be assimilated to a small drop of quark matter. Concerning the Coulomb interaction, it mainly plays a role in determining the size of the structures while it is not so important for the response of the structures to the perturbation, at least for relatively large values of the surface tension $\sigma \gtrsim 10 \text{ MeV/fm}^2$. In that case, in fact, the screening due to electrons almost completely cancels the effect of the Coulomb interaction in the nucleation rate [24]. In the following for simplicity we will assume that $10 \text{ MeV/fm}^2 \lesssim \sigma \lesssim 30 \text{ MeV/fm}^2$. In conclusion we reasonably assume that large values of σ suppress all these processes, therefore the equations describing the melting process as a fast reaction, i.e. Eq. (10) (first scenario) and Eqs. (23), (24) (second scenario) are not imposed. We can compute the viscosity by requiring that the baryon number and the electric charge of the two phases are separately conserved. This implies that Eq. (6) in the first scenario and Eqs. (20), (21) in the second scenario, each of them separate into two distinct equations. In the first scenario, Eq. (7) is not modified due to the charge neutrality of the CFL quark phase. Finally, the equation of mechanical equilibrium (Eq. (8) in the first scenario and Eq. (22) in the second scenario) is still valid and it represents the only fast process connecting the two phases. Notice that since melting processes are suppressed, the only reaction which allows the system to rapidly re-equilibrate is elastic scattering between the two phases. In the following we have assumed that a residual interaction between the two phases always exists, similarly to the existence of “entrainment” in superfluid neutron matter as discussed e.g. in Ref.[25]. The systems of equations read therefore:

$$0 = (1 - \chi)(\delta\rho_n + \delta\rho_p + \delta\rho_\Lambda + \delta\rho_\Sigma) - \delta\chi(\rho_n + \rho_p + \rho_\Lambda + \rho_\Sigma), \quad (25)$$

$$0 = \chi\delta\rho_q + \delta\chi\rho_q, \quad (26)$$

$$0 = (1 - \chi)(\delta\rho_p - \delta\rho_\Sigma) - \delta\chi(\rho_p - \rho_\Sigma), \quad (27)$$

$$0 = \sum_{\{H\}} p_H \delta\rho_H - p_q \delta\rho_q, \quad (28)$$

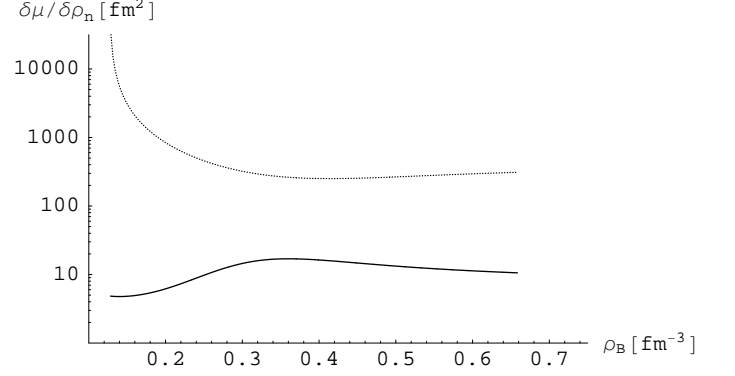


FIG. 6: The chemical unbalance for the second scenario is shown as a function of the baryonic density. The solid line corresponds to the case of a vanishing of the surface tension and the dotted line corresponds to a finite surface tension.

$$0 = \beta_n \delta\rho_n + \beta_p \delta\rho_p + \beta_\Lambda \delta\rho_\Lambda + \beta_\Sigma \delta\rho_\Sigma \quad (29)$$

for the first scenario and

$$0 = (1 - \chi)(\delta\rho_n + \delta\rho_p) - \delta\chi(\rho_n + \rho_p), \quad (30)$$

$$0 = \chi(\delta\rho_u + \delta\rho_d + \delta\rho_s) + \delta\chi(\rho_u + \rho_d + \rho_s), \quad (31)$$

$$0 = (1 - \chi)\delta\rho_p - \delta\chi\rho_p, \quad (32)$$

$$0 = \chi(2\delta\rho_u - \delta\rho_d - \delta\rho_s) + \delta\chi(2\rho_u - \rho_d - \rho_s) \quad (33)$$

$$0 = \sum_{\{H\}} p_H \delta\rho_H - \sum_{\{Q\}} p_Q \delta\rho_Q \quad (34)$$

for the second scenario.

In Fig. 3 and 4 we show the effect of a non-vanishing surface tension on the viscosity (dotted lines). On rather general grounds, one can expect that the effect of a non-vanishing surface tension is to reduce the viscosity. Indeed, the surface tension suppresses the fast processes of “melting” which are responsible for the reduction of the chemical unbalance. In particular, the only equation connecting the two phases is now the equation corresponding to the mechanical equilibrium. Moreover, in the first scenario the number of constraints on $\delta\rho_H$ reduces from four to three and, in the second scenario, the constraints on $\delta\rho_Q$ from four to two. It is also interesting to notice that in both scenarios, near the beginning of the MP ($\chi \rightarrow 0$), baryon number conservation requires $\delta\chi \rightarrow 0$ if the surface tension is non-vanishing (see Eqs. (26) and (31)). On the other hand, in the first scenario $\rho_\Lambda = 0$ at the beginning of the MP below the Λ production threshold, and therefore, the constraint $\delta\chi = 0$ is also satisfied in the absence of surface tension, as explained in footnote [32]. Therefore the dotted lines coincide with the solid lines in both panels of Fig. 3 for $\rho_B \lesssim 0.6 \text{ fm}^{-3}$.

In the second scenario, the effect of the surface tension is more dramatic. In Fig. 6 we display the chem-

ical unbalances corresponding to a vanishing and to a finite value of the surface tension, respectively. As already remarked $\delta\mu/\delta\rho_n$ is larger in presence of a surface tension. Concerning the singular behavior of the chemical unbalance near the first critical density, it stems from neglecting $\delta\rho_e$ in the equation of the electric charge conservation. While in general this is a safe approximation due to the slowness of the modified Urca process (see Ref. [8]), in this particular case this approximation implies the vanishing of the viscosity at threshold. Actually, the existence of a finite value for $\delta\rho_e$ implies that the viscosity is small but finite at threshold. It is not possible to apply directly the formalism of Ref. [8] if two *independent* perturbations ($\delta\rho_n$ and $\delta\rho_e$) exist in the system. We are therefore forced to discuss separately the viscosities stemming from the two independent unbalances. Concerning the viscosity associated with the modified Urca process, we remind that it scales as T^6 and therefore it is essentially negligible below $T \sim 10^{10}$ K while it will be included in the calculation of the stability of the star presented in the next section. In conclusion, the result corresponding to the dotted line of Fig. 4 would not be significantly modified by taking into account a finite value of $\delta\rho_e$.

IV. STABILITY OF HYBRID STARS

We can now address the problem of the stability of a rotating compact star. To compute the critical angular velocity we use the standard formalism of Refs.[4, 8, 26]. We need first to integrate the viscosity on the structure of the star, which is obtained by solving Tolman-Oppenheimer-Volkov equation. In Fig. 5 we show the structure of a $M = 1.46M_\odot$ star for the two scenarios discussed above. For simplicity we computed the structure of the star assuming a negligible value for σ . The main effect of the presence of finite size structures is to reduce the volume occupied by the MP. For $\sigma \lesssim 30$ MeV/fm², which is the limit of validity of our approach, the shrinking of the MP is rather modest [21]. The critical angular velocity Ω_{crit} is the one for which the imaginary part of the r-mode frequency vanishes and it is obtained by solving the equation

$$-1/\tau_{GR} + 1/\tau_B + 1/\tau_{B(Urca)} = 0. \quad (35)$$

Here τ_{GR} is the time scale for gravitational waves emission while τ_B and $\tau_{B(Urca)}$ are the time scales of the bulk viscosity produced by hadronic processes and by the modified Urca process of the nucleons[3], respectively. Results for the critical angular velocity are shown in Fig. 7. In the upper panel we compare the stability of a purely hyperonic star with the stability of a HyS containing hyperon-quark MP. As it can be seen, due to the large viscosity of the MP the HyS is as stable as the hyperonic star. Let us stress again that our result indicate that the viscosity in the MP is almost independent

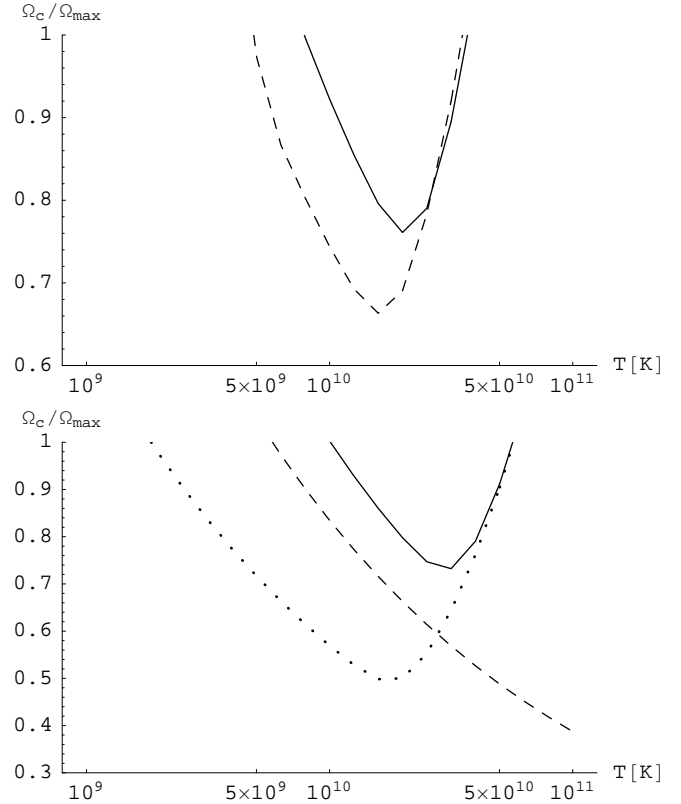


FIG. 7: Critical angular velocities. The solid lines refer to hybrid stars, dashed lines correspond to hyperonic stars (upper panel) and to strange stars (lower panel). The dotted line corresponds to the case of a large hadron-quark surface tension ($10 \text{ MeV/fm}^2 < \sigma < 30 \text{ MeV/fm}^2$, see text for details).

on the hyperonic content and therefore hybrid hyperon-quark stars can be stable as long as a tiny fraction of hyperon is present. In the lower panel we compare the stability of a star made entirely of non-interacting quarks with a HyS containing a nucleon-quark MP. We have assumed that the viscosity of the pure quark matter phase in the HyS vanishes, to simulate a CFL core. As it can be seen, the small MP region, located near the edge of the star, is sufficient to damp the r-modes. For simplicity, we have assumed quarks to be unpaired in the MP, but similar results should be obtainable if a 2SC phase is present. A feature of r-modes is that they are active mostly in the outer regions of the star, and therefore the value of the bulk viscosity at a not too large densities is crucial for the stability of the star. In particular, in the first scenario the stability of the star is due to the presence of the Σ hyperons in the pure hadronic phase and in the MP. Λ hyperons, which are produced at larger densities, play a lesser role. In the second scenario, a small window of MP, present in the outer region, is sufficient to stabilize the star. Let us also remark that the star is stable at

large temperatures, due to the modified Urca processes active in the crust. These processes does not exist in the case of pure quark stars, which are therefore unstable at large temperatures. Finally, when a finite value for the hadron-quark surface tension is taken into account

the instability window is larger, but the main conclusion concerning the stability of a young hybrid star remains valid.

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 - [29] Also the presence of a magnetic field inside the compact star can damp the r-modes on a time-scale of order hours or days [27].
 - [30] The structure of the MP obtained imposing Gibbs condition is clearly different from a two-fluid model like the one adopted e.g. in Ref. [25], in which the two phases are essentially independent
 - [31] We assume, as in Ref.[8], that all leptonic reaction rates are much smaller than those associated with non-leptonic reactions, so we do not include them in the calculation of bulk viscosity of the MP. The viscosity due to semi-leptonic reactions (modified Urca processes) becomes relevant at very high temperatures and we will include its effect when computing the stability of the star.
 - [32] If the Σ density vanishes, the corresponding density fluctuation is identically zero and the constraint given by Eq. (3) has not to be imposed. If the Λ density vanishes, then $\delta\rho_\Lambda = 0$, Eq. (3) has not to be imposed, the melting process described by Eq. (4) does not exist and $\delta\chi = 0$.
 - [33] Notice that the Coulomb interaction does not play any direct role when computing the viscosity in the present scheme, since it cannot modify the value of the chemical unbalance $\delta\mu$.
 - [34] Our results in both scenarios are consistent with the general outcome of Ref.[28] stating that in the high-frequency limit the bulk viscosity is just the sum of partial bulk viscosities in different slow reaction channels.
 - [35] Notice that if CFL gaps can form at low density then the bulk viscosity of MP vanishes.
 - [36] The reaction rate has been multiplied by a factor three, in agreement with Ref.[11].